

Parrakeet (*Pyrrhuloxia tabuan*), a Masked Parrakeet (*Pyrrhuloxia personata*) from the Fiji Islands, purchased; a Chestnut-backed Weaver-bird (*Hyphantornis castaneo-fusca*), four Rufous-necked Weaver-birds (*Hyphantornis textor*), two Grenadier Weaver-birds (*Euplectes oryx*), a Barbary Turtle dove (*Turtur risorius*), a Vinaceous Turtle dove (*Turtur vinaceus*) from West Africa, a Turquoise Parrakeet (*Euphonia pulchella*) from New South Wales, two Undulated Grass Parrakeets (*Melopsittacus undulata*), seven Crested Ground Parrakeets (*Calopsitta novaehollandiae*) from Australia, deposited; eight Mocassin Snakes (*Tropidonotus fasciatus*), born in the Gardens.

THE FIGURE AND SIZE OF THE EARTH¹ III.

DURING the years 1816-52 a Russo-Scandinavian degree-measurement was carried out by Struve and Gen. Tenner, of unusual length and wonderful accuracy. It extended from Hammerfest in the north (70° 40') to Ismail in the south (45° 20'), a length of 25° 20'. From this it followed that for latitude 56° 3' 56", the degree length is 57,137 toises, and in this was included a Swedish degree-measurement which gave 57,209 toises as the length of a degree in lat. 66° 20' 12". At this time extensions of earlier operations were undertaken in other countries; as in England with the result of $\frac{1}{288}$ for the earth's oblateness. Everest extended Lambton's East Indian degree-measurement, which at present has a length of over twenty-one degrees. Also a longitude measurement begun at an earlier period in Central Europe was recommenced and reaches now from Brest, Paris, Strassburg, Munich, to Vienna. With the assistance of the new data the amount of the earth's oblateness was again investigated, and now that there was no doubt of the accuracy of the measurements, it was shown distinctly that the earth was not an entirely regular elliptical spheroid, that the flattening did not pass regularly over the earth's surface. The theory was next propounded that the earth was an ellipsoid of three axes, but the proposition was not fully supported by the measurements.² A newly elaborated mathematical method, by which from the existing measurements, the figure of the earth could be obtained, and by which the remaining errors could be reduced to a minimum, was now applied by the great Bessel to the data before him, and according to these principles and on the basis of the best measurements, he obtained dimensions of the earth which still form the ground of all astronomical and geodetical calculations, and which are as follows:—He found for the equatorial radius of the earth 3272077.14 toises = 6377400 metres; for the polar radius 3261139.33 toises = 6356080 metres; and for the length of the earth's quadrant 10000855.765 metres; while he gave the earth's oblateness as $\frac{1}{298.153}$. He also de-

duced formulæ giving the length of a degree or of a parallel of latitude for any part of the earth; these formulæ will be found in most geodetic handbooks. New values for the dimensions of the earth were, ten years ago, deduced by Leverrier with the assistance of other measurements, but they differ very little from those of Bessel, and in almost all scientific works Bessel's constants are adopted. Thus, about the middle of the present century the solution of the original problem has been attained. And yet since that time there has unmistakably been an altogether unusual progress in all departments concerned in the solution of the problem. First of all, mechanical precision has in the last decades attained such perfection, that now astronomical observations, and especially the geographical determination of places, can be made with considerably greater accuracy than was possible a few decades ago. Moreover, these improvements have been accompanied with extraordinarily suitable new methods. The telegraphic determination of longitude, a method for the determination of the lengths of parallels, has above all, especially in the last ten years, attained such perfection, theoretically and practically, that the measurements carried out on this method are most accurate, and we may justly expect in the future most brilliant results from it. Altogether the problem is near a more substantial solution now than it ever was before, as there are so many well-equipped observa-

tories at work, bound together by a network of the most accurate determination of longitude and latitude, extended over the surface of the earth, and the completion of which is yet going on. Of the greatest importance also in this connection is the more recent theory of measurement, according to which the points to be connected need not lie in one and the same meridian or parallel, but, on the contrary, are connected by an arc of a great circle lying in any direction—the geodetic line.

Thus have we seen how from the remotest times, among all the nations of the earth not only has the wish to obtain a knowledge of the form and size of our dwelling-place prevailed, but also the most earnest activity in attaining this knowledge of nature; how this has in an increasing degree gone hand in hand with the progress of the mathematical and physical sciences, having at last reached a worthy solution in our time; and we have the best reasons for believing that, closely following the progress of the sciences, our knowledge in this connection will soon reach the highest degree of certainty.

Remarks on Karl Maria Friedrich's Paper, by Col. Clarke, R.E.

One feature of modern geodesy that has been left unnoticed by the writer of the foregoing paper is the application of the theory of probabilities, under the form of the "method of least squares," to the reduction of the observations. In any network of triangulation if it were required to obtain the distance apart of two of the extreme points, we should obtain varying results according to the set of triangles used in the calculation, and the question arises, What is the degree of credibility to be given to each result, or what the "probable error" of each? This implies, of course, that there is a superfluity of observations, that is, more than are absolutely necessary to fix all the points. If there were no superfluous observations there could be but one result obtained—an advantage in one respect (in saving calculation)—but then we should be wholly at sea as to the degree of trust to be placed in this one result. The theory of probabilities teaches us how to treat superfluous observations, and practically it leads to the calculation of a system of corrections to all the observations in order to bring them into harmony. Now there could be found an infinite variety of systems of corrections that would harmonise the observations, but the particular system we are led to by the method of least squares has this feature, that the sum of the squares of the corrections is an absolute minimum. In plain words, the triangulation is harmonised with the least possible alteration of the original observations as a whole.

These calculations, however, involve an enormous amount of labour, and one is compelled sometimes to relax the rigour of the theory, and accept some slight modifications. For example, in the triangulation of Great Britain and Ireland this method required the solution of an equation of 920 unknown quantities; this hopeless task was evaded by breaking up the triangulation into some fifteen or more parts.

The principal triangulation of India—unlike that which uniformly covers the face of this country—consists of chains of triangles running in a meridian direction crossed by several other chains perpendicular to them; as the longitudinal series from Kurrachee to Calcutta, and that from Bombay to Vizagapatam, the north-east longitudinal series, and others. The reduction of these chains of triangles by the method of least squares is a vast labour not yet completed.

The enormous mass of the Himalayan Mountains causes, or rather might theoretically be supposed to cause, an immense disturbance of the direction of gravity at the stations of the Indian arc amounting at the northern stations to 30", with smaller amounts as the distances of stations southwards increase. Now it is very singular that, except at the very foot of the mountains, these discrepancies do not actually appear (to anything like that amount) in the arc itself or when it is used in the problem of the figure of the earth. This very remarkable circumstance has led to the hypothesis that the attraction of the superincumbent visible mass must be compensated for by a diminution of density in the strata underlying the mountains. Pendulum observations made for the purpose of elucidating this point have shown conclusively that the density of these underlying strata is actually less there than elsewhere. A compensation therefore takes place—but in a general way only; it cannot be shown that it is a mathematically exact compensation, consequently there is some residual doubt as to what really are the deflections of the vertical at the astronomical stations of the Indian arc.

We may supplement Karl Maria Friedrich's account of how to calculate a meridian arc by a description of the method fol-

¹ Continued from p. 580.

² This is scarcely correct: the figure of three unequal axes agrees better with the observations than does the spheroid of revolution. But there is a necessity for this, and the ellipsoidal figure cannot be regarded as established.

lowed in calculating a portion of the English arc where proceeding from the Yorkshire coast between Redcar and Whitby (the survey station is Easington) it crosses the sea in a line to Saxaford in Shetland. At Easington astronomical observations gave the azimuth of Cheviot as $38^{\circ} 48' 58'' \cdot 68$. This determines the meridian; and the triangulation gives the relative distances and azimuths at the following succession of stations (selected for this calculation):—Cheviot; Mount Battock, on the Grampians between Aberdeen and Balmoral; Scarabin in Caithness, Fitty Hill in the Orkneys; Foula, a precipitous island between Orkney and Shetland; and Yell, a station in Shetland (not quite so far north as Saxaford (see Fig. 3).

On the meridian line defined above, take a point, A, whose distance from Easington is equal to the side "Easington to Cheviot." Join A with Cheviot and Mount Battock, and it is evident that we can determine the distance A to Mount Battock and the angle at A between Mount Battock and the north

meridian. Next take the point D at a distance from A equal to the side "A to Mount Battock." Join D with Mount Battock and Scarabin, then we can determine the side "D to Scarabin" and its inclination to the meridian at D. Next take a point G (still in the same meridian) at a distance from D equal to the side "D to Scarabin," and join G with Scarabin, Fitty Hill, and Foula. Next take H at a distance from G equal to the side "G to Foula," and join H with Foula, Yell, and Saxaford. From Saxaford drop a perpendicular on the meridian meeting it in S; then we have the distance Easington to S, and the length of this perpendicular, which is about 200 feet. In order to verify this result a different set of stations were chosen for a second calculation conducted in a similar manner; the distance Easington to S by the first calculation was 2288427.29 feet; by the second it was 2288427.38 feet.

The volume of the Ordnance Survey entitled "Account of the Principal Triangulation of Great Britain and Ireland," by Capt. A. R. Clarke, R.E., contains (pp. 733-778) an elaborate calculation of the figure of the earth based upon the English and French arcs of eleven and twelve degrees respectively, the Russian and Indian of twenty-five

degrees and twenty-one respectively, the first Indian arc, the Danish and the Prussian arcs of about a degree and a half each, the Peruvian arc of three degrees, and the Hanoverian arc of two degrees—a total length of eighty degrees within a few minutes. The figure is first investigated without restricting it to the elliptic form, but assuming that the radius of curvature of the meridian is expressed by the formula $\rho = A + 2B \cos 2\phi + 2C \cos 4\phi$ (the ellipse is a particular case of this curve, i.e., if $5B^2 - 6AC = 0$). The resulting semi-axes are $a = 20927197$ ft., $b = 20855493$ ft., $a : b = 291.9 : 290.9$, and the meridian curve is more protuberant than an ellipse of the same axes by the quantity $\delta r = (177.5 \pm 70.9) \sin^2 2\phi$. But when the curve is restricted to the form of an ellipse the semi-axes are found to be 20926348 and 20855233 with the ratio of $294.26 : 293.26$. The probable errors of the semi-axes so determined are ± 186 and ± 239 respectively.

Subsequently to the publication of the volume containing these investigations, an extensive series of comparisons of standards—the geodetic standards—of length of the various countries in which meridian arcs have been measured, was made at the Ordnance Survey Office, Southampton. In order that these most important comparisons might be made with the utmost attainable accuracy, a building was erected especially for this purpose. The comparison-room—itsself double-walled—is surrounded and wholly inclosed within an outer building, and it is partly sunk below the level of the ground. The diurnal change of temperature is not perceptible in this room, consequently comparisons are made under extremely favourable circumstances. As a rule, each pair of standards was compared in summer at about the temperatures of 62° , and in winter at about 32° or 42° , but always at the natural temperature of the room, artificial temperatures being wholly excluded. To give an idea of the precision attained in these comparisons, the probable error of the length of the Ordnance Standard ten-foot bar in terms of the national yard is four ten-millionths of a yard; that of the Indian steel standard (a ten-foot bar) is less than three ten-millionths of a yard; that of the Russian standard, a bar of more than twelve feet long (two toises) is six ten-millionths. The importance of these results, connecting the largest arcs, upon which the figure of the earth must depend, can hardly be over-estimated. The length of the "Toise of Peru" obtained through three entirely independent sources, viz., the Russian, the Prussian, and the Belgian toises, is 639453348 feet, from which the greatest divergence of the three separate results is only half a millionth of a toise; this corresponds to ten feet in the earth's radius. The length of the "metre" deduced from the above by means of its defining ratio, $(443296 : 864000)$ is 3.28086933 feet.

One of the most difficult of the determinations of length made in connection with this series was the measuring the distance between the knife edges of Kater's Reversible Pendulum. In this operation the flexibility of the bar was in the preliminary comparisons a source of much error. This, however, with some other troublesome matters, was successfully overcome, and the length was determined with a probable error of less than one five-millionth part of the length. The length of the pendulum was obtained by comparing it with the metre, and the details of the measurements will be published in the "Account of Pendulum Observations in India," by Col. Walker, R.E., F.R.S., Superintendent of the Great Trigonometrical Survey and Surveyor-General of India.

These results of the comparisons of standards of the different countries concerned, modify the numbers given above for the semi-axes of the earth. After making the necessary corrections, the semi-axes of the elliptic meridian are 20926062 feet and 20855121 feet, or 6378206.4 metres and 6356583.8 metres. The ratio of the axes $293.98 : 294.98$. See the Ordnance Survey volume, entitled "Comparisons of Standards of Length," page 287. In the same volume will be found an investigation of the figure of the earth, supposing it to be an ellipsoid of three unequal axes.

But these results are now again superseded by a more recent determination published in the *Philosophical Magazine* for August. It appears that during the last few years the Surveyor-General of India, Col. Walker, C.B., F.R.S., Royal Engineers, has been not only measuring new arcs in India, both of latitude and of longitude, but has revised the southern portion of the Indian arc as measured originally by Col. Lambton. On this chain of triangles considerable doubt rested as to what was the unit of length used in the measure; a complete remeasurement according to modern methods has set this question at rest. A complete meridian chain of triangles has been carried from Mangalore on the west coast in latitude $12^{\circ} 52'$ and longitude $75^{\circ} E.$, to a point in latitude 32° ; thus the whole Indian arc is now 24° in length. Eleven differences of longitude have been determined by electro-telegraphy between the stations Mangalore, Bombay, Vizagapatam, Madras, Bangalore, Hyderabad, and Bellary; it is almost unnecessary to add that in these operations no refinements of modern science have been overlooked. As the difference of longitudes of Bombay and Vizagapatam is $10^{\circ} 28'$ and the geodetic connections of the above seven stations is completed (liable, however, to some small future corrections resulting from the least square calculations) there is thus presented a large addition to the data of the problem of the figure of the earth. Taking the English and French conjoined arc of 22° with fifteen astronomical stations; the Russian arc of 25° with its thirteen stations; the Indian meridian arc of 24° with

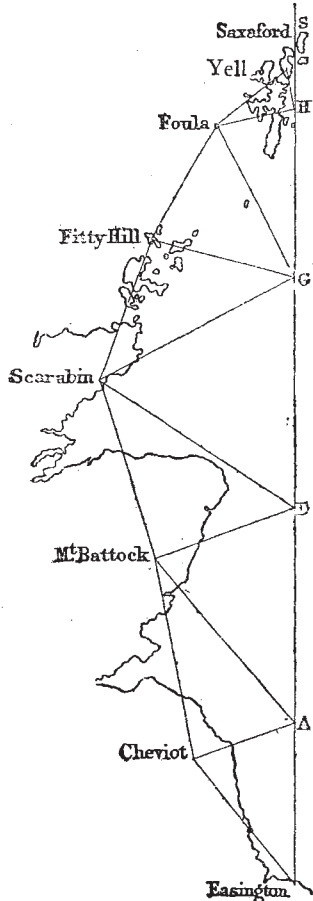


FIG. 3.

fourteen stations (this is not a fourth part of the number available), and the Indian longitude stations specified above, fifty-six equations treated by least squares give the following elements of the earth's figure:—

Polar semi-axis $c = 20854895$ standard feet.
Equatorial semi-axis $a = 20926202$ " "
 $c : a = 292'465 : 293'465$.

But the evidence of the Indian longitude observations goes to show that the curvature of the surface of India in a direction perpendicular to the meridian is considerably less than that belonging to the spheroid just specified. Possibly this apparent result may be owing to the existence of attractions of the plumb-line seawards at the coast stations. At any rate it suggests the re-investigation of the ellipsoidal form of the earth: and the result of a formidable calculation is that the ellipsoid best representing all the observations has the following semi-axes:—

$a = 20926629$ $b = 20925105$ $c = 20854477$,
and the ellipticities of the two principal meridians

$$\frac{1}{289'5} \text{ and } \frac{1}{295'8}.$$

The longitude of the greater axis of the equator is $8^{\circ} 15'$ W. of Greenwich—a meridian passing through Ireland and Portugal, and cutting off a portion of the north-west corner of Africa; in the opposite hemisphere this meridian cuts off the north-eastern corner of Asia and passes through the southern island of New Zealand. The meridian containing the smaller diameter of the equator passes through Ceylon on the one side of the earth and bisects North America on the other. Thus the division of the earth by the meridian plane of the greater axis of the equator corresponds very nearly with the ordinary two-circle representation of the earth, the one showing the Eastern hemisphere the other the Western.

Such is the result of the calculation, and it is a somewhat remarkable result when considered in connection with the actual physical features of the globe, and the distribution of land and sea on its surface. But too much confidence must not be placed in it; many more measurements would be necessary to establish this figure as a reality; as yet it is merely indicated by the existing observations, and the amount of the eccentricity of the equator shown above is really very minute.

It is to be observed—returning to the spheroidal figure and comparing this new result with that quoted above from the volume of "Comparisons of Standards"—that the effect of the new work in India has been to increase the radius of the equator by 140 feet, and to diminish the polar radius by 226 feet.

There are several short arcs on the European continent which might have been used in addition to the long arcs, but the influence of these on the result would have been almost imperceptible. The details of the American Coast Survey oblique arc are not yet published.

Notwithstanding the immense additions to geodetical measurements and to the data of the problem of the figure of the earth since Bessel's investigations (1841), it is with a good deal of truth that Karl Maria Friederici says that Bessel's results are still universally adopted by scientific men. And this must be considered a very remarkable instance of the influence of a name. Bessel was a splendid mathematician; his works are characterised by great elegance; and in this case his fame is a set-off against the increase of data subsequent to his time. But Bessel could only use one-fourth part of the present English arc (and the terrestrial measure of this arc as used by him was some 200 feet in error), and one-third of the present Russian arc. In his time the English, Russian, and Indian arcs amounted in all to less than 27° ; now they exceed 57° . Hence Bessel's figure of the earth cannot be considered anything else than obsolete, however excellent it may have been six-and-thirty years ago.

The operations which are being conducted with so much activity on the European continent and in India must shortly put us in possession of great additions to the data of the problem, especially through the agency of the electric telegraph. As a specimen of the precision now attainable in the determination of longitudes by galvanic signals, we may quote the three results obtained at different times and in different ways for the difference of longitude of Greenwich Observatory and Harvard Observatory (Cambridge, Massachusetts). They are as follows:—

	h.	m.	s.
In 1866, by Anglo-American cables	4	44	31'00
In 1870, by French cables to Duxbury	4	44	30'99
In 1872, by French cable to St. Pierre	4	44	30'96

Doubtless there is a certain amount of good fortune in this, but nevertheless the accordance is highly satisfactory.

AN EXPERIMENTAL INVESTIGATION OF THE STRUCTURE OF FLUID COLUMNS WHICH ARE AFFECTED BY SOUND

WHEN a fluid escapes from a contracted opening, it may form a column, which throughout the greater part of its length has the same sectional shape as the opening. This kind of column may be called prismatic.

It may after leaving the opening form an expansion, this expansion being succeeded by another at an angle (usually a right angle) to it; and this latter by another, and so on. This kind of column may be called segmental (Fig. 1).

An example of the first is obtained when the column proceeds from a true cylinder, truncated at right angles to its axis, and is very difficult to obtain.

A segmental column is easily obtained from the end of a partially closed glass tube.

Segmental columns are sensitive, prismatic columns not sensitive, to sound.

The character of a jet is determined by connecting with a water supply.

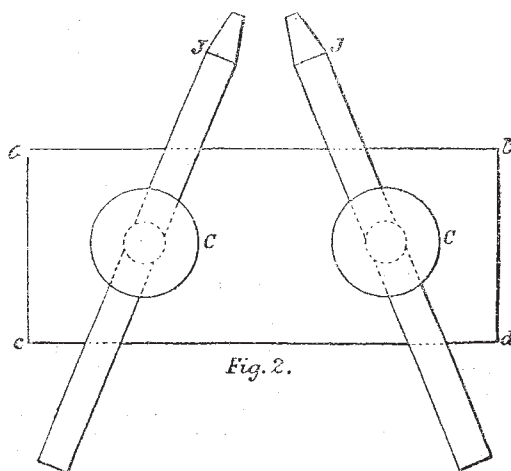
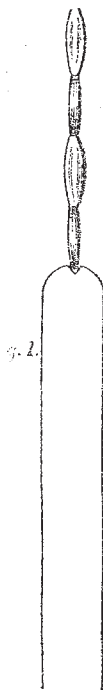
The apparatus, Fig. 2, is intended to show the structure of a segmental column. a, b, c, d is a piece of cork $2\frac{1}{2}$ in. $\times$ 1 in. and 1 inch thick. Pieces of glass tube 2 inches long serve as axes for the corks c, c . Each cork is pierced at right angles to its axis to support a jet J . An india-rubber T-piece connects the jets with gas or water supply.

It will be seen that the arrangement permits of the jets being inclined to each other at any angle by the movement of the corks round their axes; and a lateral adjustment may be obtained by sliding them along their axes.

To make the jets: the middle of a long piece of glass tube is contracted in the blowpipe flame; when cold a sharp scratch is made in the middle of the contraction, and the tube broken through. Two perfectly equal jets are thus obtained.

Experiment I.—Arrange the jets at an acute angle and connect with a water supply. The segmental column will be obtained. The first (primary) expansion being at right angles to the plane of the jets.

Experiment II.—Keeping the water supply as before, increase the angle of the jets; the segments will become more marked and the length of the entire column decreased. At a certain angle the primary expansion becomes so great that the cohesion of the



different parts will be overcome, and the column be fan-shaped. The ordinary fish-tail burner furnishes a familiar example of this extreme segmentation.

Note.—As every degree of segmentation may be obtained with suitable inclination of the jets, it follows that a segmental column really consists of two similar ones, meeting at an angle.